# New ideas on SUSY searches at future linear colliders ${ }^{\star}$ 

S. Hesselbach ${ }^{1}$, O. Kittel ${ }^{2}$, G. Moortgat-Pick ${ }^{3}$, and W. Öller ${ }^{4}$<br>${ }^{1}$ Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria<br>${ }^{2}$ Institut für Theoretische Physik und Astrophysik, Universität Würzburg, D-97074 Würzburg, Germany and Instituto de Física Corpuscular - C.S.I.C., Universitat de València, E-46071 València, Spain<br>${ }^{3}$ IPPP, University of Durham, Durham DH1 3LE, UK<br>${ }^{4}$ Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, A-1050 Vienna, Austria

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#### Abstract

Several results obtained within the SUSY group of the ECFA/DESY linear collider study are presented: (i) a possibility to determine $\tan \beta$ and the trilinear couplings $A_{f}$ via polarisation in sfermion decays, (ii) the impact of complex MSSM parameters on the third generation sfermion decays, (iii) determination of CP violation in the complex MSSM via T-odd asymmetries in neutralino production and decay, and (iv) an analysis of the chargino and neutralino mass parameters at one-loop level.


PACS. 14.80.Ly Supersymmetric partners of known particles

## 1 Polarisation in sfermion decays: Determining $\tan \beta$ and trilinear couplings

The $\mathrm{SU}(2) \times \mathrm{U}(1)$ gaugino mass parameters $M_{2}$ and $M_{1}$, as well as the higgsino mass parameter $\mu$ and $\tan \beta$ for $\tan \beta \leq 10$ can be extracted with high precision from the chargino/neutralino sector [1]. In [2] it has been shown that sfermion production is also suitable for investigating the properties of neutralinos. In case of $\tan \beta \geq 10$ it is appropriate to determine this parameter via polarisation effects in sfermion decays to fermions plus neutralinos/charginos in $e^{+} e^{-}$pair production of third-generation sfermions [3], $e^{+} e^{-} \rightarrow \tilde{f}_{i} \tilde{\tilde{f}}_{j}, \tilde{f}_{i} \rightarrow f \tilde{\chi}_{k}, \quad f=\tau, t, b$.

A simulation at one reference scenario RP with $\tan \beta=$ 20, inspired by SPS1a 4], is performed in [3]. It is possible to measure with high precision $m_{\tilde{\tau}_{1}}=154.8 \pm 0.5 \mathrm{GeV}$ in the hadronic decay spectra (see e.g. Fig. 7 in the second publication of [3]) as well as the polarisation $P_{\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{1}^{0}}=$ $0.82 \pm 0.03$. The cross section can be measured with an accuracy of $\delta \sigma / \sigma=3 \%$. The use of polarised beams leads to the unambiguous determination of the mixing angle $\cos 2 \theta_{\tilde{\tau}}=-0.987 \pm 0.08$. The inversion of the polarisation with respect to its dependence on $\tan \beta$ including the complete gaugino/higgsino mixing leads to the determination of $\tan \beta=22 \pm 2$ (Fig. (1).

An analogous procedure can be applied for $\tilde{t}$ and $\tilde{b}$ production. Since the $t$ polarisation in the process $\tilde{t}_{i} \rightarrow t \tilde{\chi}_{k}^{0}$ depends on $1 / \sin \beta$, it is only weakly sensitive to large $\tan \beta$. By contrast, the decay $\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{ \pm}$can be used indeed to measure $\tan \beta$. The top polarisation measurement requires the reconstruction of the $t$ system and of the di-

[^0]

Fig. 1. $\tan \beta$ versus $\tau$ polarisation $P_{\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{1}^{0}}$ for the reference scenario RP. The bands illustrate a measurement of $P_{\tau}=0.82 \pm 0.03$ leading to $\tan \beta=22 \pm 2$. For details see 3]
rection of the primary squark $\tilde{b}_{1}$ which can be determined up to a twofold ambiguity and leads to a measurement in the RP $P_{\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{0}}=-0.44 \pm-0.10$.

In case that the heavier $\tilde{f}_{2}$ is accessible, one could determine the trilinear coupling $A_{f}$ (for earlier studies see also [5]), $A_{f}=\left[m_{\tilde{f}_{1}}^{2}-m_{\tilde{f}_{2}}^{2}\right] / 2 m_{f} \sin 2 \theta_{\tilde{f}}+\mu\binom{\tan \beta}{\cot \beta}$.

A summary of the expected precision of $\tan \beta$ and $A_{f}$ in our reference scenario RP is given in Table 14 [3].

## 2 Third generation sfermion decays in complex MSSM

So far most phenomenological studies on production and decay of SUSY particles have been performed within the MSSM with real SUSY parameters. In 6, 7] production and decays of the third generation sfermions in the MSSM

Table 1. Summary of expected precisions of the sfermion mixing parameters $\sin 2 \theta_{\tilde{f}}, \tan \beta$ and the trilinear couplings $A_{f}$ in reference scenario RP. For details on the error estimates see [3]

| $\tilde{f}$ | $\tan \beta$ |  | $A_{f}[\mathrm{GeV}]$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | ideal | error | ideal | error |
| $\tilde{\tau}$ | 20 | 2 | -254 | - |
| $\tilde{b}$ | 20 | 4.5 | -773 | 450 |
| $\tilde{t}$ | 20 | - | -510 | 50 |



Fig. 2. Branching ratios $B\left(\tilde{t}_{1} \rightarrow \tilde{\chi}_{1}^{+} b\right)$ (solid), $B\left(\tilde{t}_{1} \rightarrow \tilde{\chi}_{1}^{0} t\right)$ (dashed) and $B\left(\tilde{t}_{1} \rightarrow \tilde{\chi}_{2}^{+} b\right)$ (dashdotted) in a scenario inspired by SPS 1a [4] for $\varphi_{A_{b}}=\varphi_{\mu}=\varphi_{M_{1}}=0, \tan \beta=10$ and $\left\{m_{\tilde{t}_{1}}\right.$, $\left.m_{\tilde{t}_{2}},\left|A_{t}\right|,|\mu|, M_{2}\right\}=\{379,575,466,352,193\} \mathrm{GeV}[7]$
with complex parameters $A_{\tau}, A_{t}, A_{b}, \mu$ and $M_{1}$ are analyzed. In a large region of the MSSM parameter space the branching ratios of $\tilde{\tau}_{1,2}, \tilde{\nu}_{\tau}, \tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ show a strong phase dependence. This could have an important impact on the search for third generation sfermions at a future linear collider and on the determination of the supersymmetric parameters.

In [6] the effects of the CP phases of $A_{\tau}, \mu$ and $M_{1}$ on production and decay of $\tilde{\tau}_{1,2}$ and $\tilde{\nu}_{\tau}$ are studied. The branching ratios of fermionic decays of $\tilde{\tau}_{1}$ and $\tilde{\nu}_{\tau}$ show a significant phase dependence for $\tan \beta \lesssim 10$ whereas it becomes less pronounced for $\tan \beta>10$. The branching ratios of the $\tilde{\tau}_{2}$ into Higgs bosons depend very sensitively on the phases for $\tan \beta \gtrsim 10$.

In [7] the impact of the CP phases of $A_{t}, A_{b}, \mu$ and $M_{1}$ on the decays of $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ are analyzed. The branching ratios of the $\tilde{t}_{1,2}$ show a pronounced phase dependence in a large region of the MSSM parameter space (Fig. 21). In the case of $\tilde{b}_{i}$ decays there can be appreciable $\varphi_{A_{b}}$ dependence, if $\tan \beta$ is large and the decays into Higgs bosons are allowed.

Further the expected accuracy in determining the supersymmetric parameters was estimated by a global fit of measured masses, branching ratios and production cross sections. $A_{\tau}, A_{t}$ and $A_{b}$ can be expected to be measured with $10 \%, 2-3 \%$ and $50 \%$ accuracy, respectively, $\tan \beta$ with $1 \%(2 \%)$ accuracy in case of small (large) $\tan \beta$ and the other parameters with $1 \%$ accuracy.


Fig. 3. Contour lines of the asymmetry $\mathcal{A}_{\mathrm{T}}$ in the $|\mu|-M_{2}$ plane for $\varphi_{M_{1}}=0.5 \pi, \varphi_{\mu}=0$, taking $\tan \beta=10, m_{0}=100$ $\mathrm{GeV}, \sqrt{s}=500 \mathrm{GeV}$ and $\left(P_{e^{-}}, P_{e^{+}}\right)=(0.8,-0.6)$. The area A (B) is kinematically forbidden since $m_{\tilde{\chi}_{1}^{0}}+m_{\tilde{\chi}_{2}^{0}}>\sqrt{s}\left(m_{\tilde{\ell}_{R}}>\right.$ $m_{\tilde{\chi}_{2}^{0}}$ ). The gray area is excluded since $m_{\tilde{\chi}_{1}^{ \pm}}<104 \mathrm{GeV}[10$

## 3 CP violation in MSSM with complex parameters

The phases $\varphi_{M_{1}}$ and $\varphi_{\mu}$ have also impact on the phenomenology of neutralino production and decay at a future linear $e^{+} e^{-}$collider and give rise to CP- and T-odd observables. Such observables, which involve triple products [8, may be large, because they already arise on tree level. In addition, they also allow the determination of the sign of the phases. In neutralino production (for recent studies see [1, 9 ):

$$
\begin{equation*}
e^{+}+e^{-} \rightarrow \tilde{\chi}_{i}^{0}+\tilde{\chi}_{j}^{0} \tag{1}
\end{equation*}
$$

and the subsequent leptonic two-body decay of one of the neutralinos and of the decay slepton

$$
\begin{equation*}
\tilde{\chi}_{i}^{0} \rightarrow \tilde{\ell}+\ell_{1}, \quad \tilde{\ell} \rightarrow \tilde{\chi}_{1}^{0}+\ell_{2}, \quad \ell_{1,2}=e, \mu, \tau \tag{2}
\end{equation*}
$$

the triple product $\mathcal{T}=\left(\mathbf{p}_{e^{-}} \times \mathbf{p}_{\ell_{2}}\right) \cdot \mathbf{p}_{\ell_{1}}$ defines the T-odd asymmetry of the cross section $\sigma$ for the processes (1), (2):

$$
\begin{equation*}
\mathcal{A}_{\mathrm{T}}=\frac{\sigma(\mathcal{T}>0)-\sigma(\mathcal{T}<0)}{\sigma(\mathcal{T}>0)+\sigma(\mathcal{T}<0)} \tag{3}
\end{equation*}
$$

In [10] the dependence of $\mathcal{A}_{\mathrm{T}}$ on $\varphi_{M_{1}}$ and $\varphi_{\mu}$ is analyzed. In Fig. $3 \mathcal{A}_{\mathrm{T}}$ is shown in the $|\mu|-M_{2}$ plane for $e^{+} e^{-} \rightarrow \tilde{\chi}_{1}^{0} \tilde{\chi}_{2}^{0}$ with the subsequent decay of $\tilde{\chi}_{2}^{0}$ into the right selectron and right smuon, $\tilde{\chi}_{2}^{0} \rightarrow \tilde{\ell}_{R} \ell_{1} . \mathcal{A}_{\mathrm{T}}$ reaches values up to $9 \%$. Choosing $\varphi_{M_{1}}=0.1 \pi$ and $\varphi_{\mu}=0, \mathcal{A}_{\mathrm{T}}$ would still reach values up to $5 \%$. A small value of $\varphi_{M_{1}}$ and in particular of $\varphi_{\mu}$ is suggested by constraints on electron and neutron electric dipole moments for a typical SUSY scale of the order of a few 100 GeV . The cross section


Fig. 4. Relative corrections to the $\tilde{\chi}_{1}^{+}$mass, fixing $M_{2}$ and $\mu$ in the chargino (full lines) and neutralino (dashed lines) sector. The parameters are $\left\{m_{A^{0}}, \tan \beta, M_{\tilde{Q}_{1}}, M_{\tilde{Q}}, A, \mu\right\}=\{500$, $40 / \mathrm{GeV}, 300,300,-400,-220\} \mathrm{GeV}$. The grey areas are excluded by the bound $m_{\tilde{\chi}_{1}^{+}} \geq 100 \mathrm{GeV}$ [14]
$\sigma=\sigma\left(e^{+} e^{-} \rightarrow \tilde{\chi}_{1}^{0} \tilde{\chi}_{2}^{0}\right) \times \operatorname{BR}\left(\tilde{\chi}_{2}^{0} \rightarrow \tilde{\ell}_{R} \ell_{1}\right) \times \operatorname{BR}\left(\tilde{\ell}_{R} \rightarrow \tilde{\chi}_{1}^{0} \ell_{2}\right)$ which is not shown, reaches values up to 60 fb . Both $\mathcal{A}_{\mathrm{T}}$ and $\sigma$ also depend sensitively on the polarizations of the $e^{+}$and $e^{-}$-beams [10].

## 4 Chargino and neutralino mass parameters at one-loop level

As mentioned in Sect. 1 the neutralino ( $\tilde{\chi}^{0}$ ) and chargino ( $\tilde{\chi}^{ \pm}$) mass parameters can be extracted at lowest order from the masses and production cross sections in $e^{+} e^{-}$ collisions with polarized beams [1]. At higher order, this extraction is not trivial and depends on the renormalization scheme. In the scale dependent $\overline{\mathrm{DR}}$ scheme the oneloop corrections to the $\tilde{\chi}^{0}$ and $\tilde{\chi}^{ \pm}$mass matrices were calculated in 11. For the on-shell renormalization various methods were proposed [12,13. They differ by different counter terms for the parameters $M_{1}, M_{2}$ and $\mu$. Although the schemes are equivalent in the sense that the observables (masses, cross sections, etc.) are the same, the meaning of the parameters are different.

Using the scheme in [12], it is shown in [14] that at one-loop level the values of the on-shell parameters $M_{2}$ and $\mu$ depend on whether they are determined from the $\tilde{\chi}^{0}$ or $\tilde{\chi}^{ \pm}$system (see Fig. (4), while the $\overline{\mathrm{DR}}$ parameters are equal in both sectors. Assuming the $\mathrm{SU}(5)$ GUT relation for the $\overline{\mathrm{DR}}$ gaugino mass parameters, we obtain a finite shift for the on-shell values $M_{1}=\frac{5}{3} \tan ^{2} \theta_{W} M_{2}+\Delta Y_{11}$. In such a way it is possible to test the GUT relation (see Fig. (5).

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Fig. 5. The ratio $M_{1} / M_{2}$ as a function of the $\tilde{\chi}_{2}^{+}$mass. The full, dashed and dotted line corresponds to the $\overline{\mathrm{DR}}$, on-shell 12 and effective [13] parameters. $\left\{m_{\tilde{\chi}_{1}^{+}}, m_{\tilde{\chi}_{1}^{0}}, \tan \beta, m_{A^{0}}, M_{\tilde{Q}_{1}}\right.$, $\left.M_{\tilde{Q}}, A\right\}=\{135,120,20 / \mathrm{GeV}, 600,350,350,500\} \mathrm{GeV}$ [14]

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[^0]:    * Talk presented by S. Hesselbach

